Quark correlations and single spin asymmetries

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We analyze the Sivers asymmetry in the light-cone gauge. The average transverse momentum of the quark distribution is related to the correlation between the quark distribution and the transverse component of the gauge field at $x^- = \pm\infty$. We then use finiteness conditions for the light-cone Hamiltonian to relate the transverse gauge field at $x^- = \pm\infty$ to the color density integrated over x^-. This result allows us to relate the average transverse momentum of the active quark to color charge correlations in the transverse plane.

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I. INTRODUCTION

Many high energy inclusive hadron processes show surprisingly large transverse polarizations or single-spin asymmetries (SSAs) [1]. The most prominent example is inclusive hyperon production, but similar phenomena are observed in many other reactions as well. For example, in the inclusive photoproduction of pions on a transversely (relative to the photon momentum) polarized nucleon target, a left-right asymmetry of the produced pions is observed [2]. Two mechanisms (which are not exclusive) have been identified as a potential source of the asymmetry: the Sivers and the Collins mechanisms. In the Collins mechanism [3], the asymmetry arises when a transversely polarized quark fragments into pions with a left-right asymmetry. In contrast, in the Sivers mechanism [4] the asymmetry results from an intrinsic transverse momentum asymmetry of the quarks in the target nucleon. At first, such an intrinsic transverse momentum asymmetry was expected to vanish due to time-reversal invariance of the strong interaction. However, more recently it was realized that, even at high energies, the final state interactions (FSI) of the struck quark play an important role for the single-spin asymmetry [5]. Formally the FSI can be taken into account by introducing an appropriate Wilson line gauge link along the trajectory of the ejected quark [6,13]. The gauge invariantly defined transverse momentum distributions with a gauge link along the light cone to infinity are no longer required to vanish due to time-reversal invariance and a nonzero Sivers asymmetry is possible.

However, while the above reasoning explains the existence of the Sivers asymmetry it leaves many questions unanswered: for example, what sign and what magnitude should one expect for the asymmetry, i.e., is it just an obscure small effect or is it large? If there is a large asymmetry, what does any information about the asymmetry teach us about the structure of the nucleon? In fact, it is also possible that there is no simple connection between the asymmetry and ground state properties of the nucleon since the asymmetry hinges on the inclusion of FSI. In this paper an attempt will be made to make a step toward answering these questions.

The paper is organized as follows. In Sec. II, we review the definitions of gauge invariant transverse momentum distributions and the role of the gauge field at $x^- = \pm\infty$ in the light-cone gauge. In Sec. III, we use finiteness constraints for light-cone Hamiltonians to derive operator constraints that allow one to relate the gauge fields at $x^- = \pm\infty$ to degrees of freedom at finite x^-. In Sec. IV we use these operator constraints to relate the average transverse momentum to color charge correlations in the transverse plane.

II. INITIAL (FINAL) STATE INTERACTIONS AND TRANSVERSE SPIN ASYMMETRIES

Reference [6] explains how final state interactions and initial state interactions (ISI) allow the existence of T-odd parton distribution functions. Formally the FSI (ISI) can be incorporated into $\mathbf{k}_\perp$ dependent parton distribution functions (PDFs) by introducing a gauge string from each quark field operator to infinity [6]:

$$q(x,\mathbf{k}_\perp,\mathbf{s}_\perp) = \int \frac{dy^- d^2\mathbf{y}_\perp}{16\pi^3} e^{-ixp^+y^- + i\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \times\langle p|\overline{q}(y^-,\mathbf{y}_\perp)\gamma^+[y^-,\mathbf{y}_\perp;\infty^-,\mathbf{y}_\perp] \times[\infty^-,\mathbf{0}_\perp;0^-,\mathbf{0}_\perp]q(0)|p\rangle. \tag{1}$$

We use light-front (LF) coordinates, which are defined as $y^\mu = (y^+,y^-,\mathbf{y}_\perp)$, with $y^\pm = (y^0 \pm y^3)/\sqrt{2}$. In all correlation functions, $y^+=0$ and we therefore do not explicitly show the y^+ dependence. The path ordered Wilson-line operator from the point y to infinity is defined as

$$[\infty^-,\mathbf{y}_\perp;y^-,\mathbf{y}_\perp] = P\ \exp\left(-ig\int_{y^-}^{\infty} dz^- A^+(y^+,z^-,\mathbf{y}_\perp)\right). \tag{2}$$

The specific choice of path in Eq. (1) reflects the FSI (ISI) of the active quark in an eikonal approximation. The complex phase in Eq. (1) is reversed under time reversal and therefore T-odd PDFs may exist [6], which is why a nonzero Sivers asymmetry [4] is possible.

Naively, the single spin asymmetry seems to be absent in the light-cone gauge $A^+=0$, since the Wilson lines in Eq. (1) are in the x^- direction and therefore $\int dz^- A^+ = 0$. Without the phase factor any single spin asymmetry vanishes due to time reversal invariance. This apparent puzzle has been resolved in Ref. [7], where it has been emphasized that a truly gauge invariant definition for unintegrated parton den-

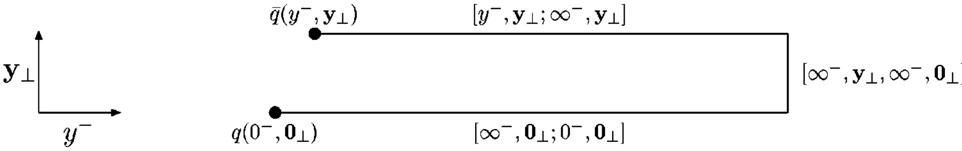

FIG. 1. Illustration of the gauge links in Eq. (3).

sities requires closing the gauge link at $x^-=\infty$, i.e. a fully gauge invariant version of Eq. (1) reads (see Fig. 1)

$$q(x,\mathbf{k}_\perp,\mathbf{s}_\perp)=\int\frac{dy^-d^2\mathbf{y}_\perp}{16\pi^3}e^{-ixp^+y^-+i\mathbf{k}_\perp\cdot\mathbf{y}_\perp}\times\langle p|\overline{q}(y^-,\mathbf{y}_\perp)\gamma^+[y^-,\mathbf{y}_\perp;\infty^-,\mathbf{y}_\perp]\times[\infty^-,\mathbf{y}_\perp,\infty^-,\mathbf{0}_\perp][\infty^-,\mathbf{0}_\perp;0^-,\mathbf{0}_\perp]q(0)|p\rangle. \tag{3}$$

In all commonly used gauges, except the light-cone gauge, the gauge link at $x^-=\infty$ is not expected to contribute to the matrix element, since the gauge fields are expected to fall off rapidly enough at ∞. However, this is not true in the light-cone gauge and therefore it has been suggested in Ref. [7] that, in the light-cone gauge, the entire single-spin asymmetry arises from the phase due to the gauge link at $x^-=\infty$ (here and in the rest of this paper we will work in the light-cone gauge $A^+=0$):

$$q(x,\mathbf{k}_\perp,\mathbf{s}_\perp)=\int\frac{dy^-d^2\mathbf{y}_\perp}{16\pi^3}e^{-ixp^+y^-+i\mathbf{k}_\perp\cdot\mathbf{y}_\perp}\langle p|\overline{q}(y^-,\mathbf{y}_\perp)\times[\infty^-,\mathbf{y}_\perp,\infty^-,\mathbf{0}_\perp]\gamma^+q(0)|p\rangle. \tag{4}$$

This implies for the average transverse momentum for quark flavor q

$$\int d^2\mathbf{k}_\perp q(x,\mathbf{k}_\perp,\mathbf{s}_\perp)\mathbf{k}_\perp=-g\int\frac{dy^-}{4\pi}e^{-ixp^+y^-}\langle p|\overline{q}(y^-,\mathbf{0}_\perp)\gamma^+\frac{\lambda_a}{2}\times\mathbf{A}_{\perp\mathbf{a}}(\infty^-,\mathbf{0}_\perp)q(0)|p\rangle. \tag{5}$$

A similar result holds for the unintegrated parton density $q_{past}(x,\mathbf{k}_\perp,\mathbf{s}_\perp)$ relevant for the Drell-Yan process, where the initial state interaction is incorporated into analogous definitions with a past-pointing gauge link. In particular, in the light-cone gauge, the asymmetry $\int d^2\mathbf{k}_\perp q_{past}(x,\mathbf{k}_\perp,\mathbf{s}_\perp)\mathbf{k}_\perp$ satisfies an analogous relation with the gauge field in Eq. (5) replaced by $\mathbf{A}_{\perp\mathbf{a}}(-\infty^-,\mathbf{0}_\perp)$.

The fact that these asymmetries hinge on the value of the transverse gauge field at $x^-=\pm\infty$ makes the evaluation of these matrix elements rather tricky. Only a careful regularization prescription for the $k^+=0$ singularity of the gauge field propagator is capable of generating the complex phase that is necessary for a non-vanishing SSA. Therefore, the question arises whether knowledge of the light-cone wave function for the nucleon would (in principle) contain sufficient information to calculate the SSA nonperturbatively, since the abovementioned prescription is perturbatively defined.

One of the main goals of this paper is to render Eq. (5) into a more useful form. For this purpose we first use the fact that the asymmetric part of the unintegrated parton densities relevant for semi-inclusive deep inelastic scattering DIS and Drell-Yan are equal and opposite [6], i.e. $\int d^2\mathbf{k}_\perp q(x,\mathbf{k}_\perp,\mathbf{s}_\perp)\mathbf{k}_\perp=-\int d^2\mathbf{k}_\perp q_{past}(x,\mathbf{k}_\perp,\mathbf{s}_\perp)\mathbf{k}_\perp$, and therefore

$$\begin{aligned}\overline{\mathbf{k}_\perp}_q(x)&\equiv\int d^2\mathbf{k}_\perp q(x,\mathbf{k}_\perp,\mathbf{s}_\perp)\mathbf{k}_\perp\\&=\frac{1}{2}\left[\int d^2\mathbf{k}_\perp q(x,\mathbf{k}_\perp,\mathbf{s}_\perp)\mathbf{k}_\perp-\int d^2\mathbf{k}_\perp q_{past}(x,\mathbf{k}_\perp,\mathbf{s}_\perp)\mathbf{k}_\perp\right]\\&=-\frac{g}{2}\int\frac{dy^-}{4\pi}e^{-ixp^+y^-}\times\langle p|\overline{q}(y^-,\mathbf{0}_\perp)\gamma^+\frac{\lambda_a}{2}\boldsymbol{\alpha}_{\perp a}(\mathbf{0}_\perp)q(0)|p\rangle,\end{aligned}$$

where $\boldsymbol{\alpha}_{\perp a}(\mathbf{0}_\perp)\equiv\mathbf{A}_{\perp\mathbf{a}}(\infty^-,\mathbf{0}_\perp)-\mathbf{A}_{\perp\mathbf{a}}(-\infty^-,\mathbf{0}_\perp)$. Equation (6) still involves the gauge field at $x^-=\pm\infty$. In the following section, we will derive an operator relation that relates $\mathbf{A}_\perp(\pm\infty,\mathbf{y}_\perp)$ to degrees of freedom at $-\infty<x^-<\infty$.

III. FINITENESS CONDITIONS

The requirement that the light-cone energy is finite (does not have infrared divergences) at $x^-=\pm\infty^-$ implies that the gauge field tensor $\mathcal{F}_a^{\mu\nu}$ vanishes at $x^-=\pm\infty^-$. This requirement places several constraints on $\boldsymbol{\alpha}_{\perp a}(\mathbf{x}_\perp)$. To see this, we start from the $+$ component of the QCD equations of motion in the light-cone gauge $A^+=0$ [8,9,11],

$$D_\mu\mathcal{F}_a^{\mu+}\equiv\partial_-\mathcal{F}_a^{-+}+\partial^i\mathcal{F}_a^{i+}-gf_{abc}A_b^i\mathcal{F}_c^{i+}=j_a^+ \tag{6}$$

where (since $A^+=0$) $\mathcal{F}_a^{-+}=-\partial_-A_a^-$ as well as $\mathcal{F}_a^{i+}=-\partial_-A_a^i$ and $j_a^+\equiv g\Sigma_q\overline{q}\gamma^+(\lambda_a/2)q$ is the fermion color density. Written out in components the field equations (6) thus read

$$-\partial_-^2A_a^--\partial_-\partial^iA_a^i-gf_{abc}A_b^i\mathcal{F}_c^{i+}=j_a^+. \tag{7}$$

Integrating Eq. (7) over x^-, while making use of the condition that the field strength tensor F_a^{+-} at $x^-=\pm\infty$ vanishes, yields

$$\partial^i \alpha_a^i(\mathbf{x}_\perp) = -\rho_a(\mathbf{x}_\perp) \equiv -\int dx^- J_a(x^-,\mathbf{x}_\perp), \tag{8}$$

$$J_a(x^-,\mathbf{x}_\perp) = -g f_{abc} A_b^i \partial_- A_c^i + g \sum_q \bar{q}\gamma^+ \frac{\lambda_a}{2} q. \tag{9}$$

Further constraints on the hadron Fock state result from the requirement that such divergences are also absent in the fermion kinetic energy term, which leads to the so-called ladder relations [10], but we will not pursue these ladder relations any further here.

An additional condition arises from the requirement that the $\mathrm{tr}[(\mathcal{F}_{12})^2]$ term in the Hamiltonian is convergent at $x^- = \pm\infty$: the field strength tensor $\mathcal{F}_{12}(x^-,\mathbf{x}_\perp)=0$ must vanish at $x^- = \pm\infty$, i.e. $A^j(\pm\infty^-,\mathbf{x}_\perp)$ must be pure gauge [7]. This allows us to gauge transform $A^j(-\infty^-,\mathbf{x}_\perp)$ to zero while preserving $A^+=0$. Since in this gauge $A^j(+\infty^-,\mathbf{x}_\perp)$ is still pure gauge, i.e. $A^j(\infty^-,\mathbf{x}_\perp) = -(i/g)U^\dagger(\mathbf{x}_\perp)\partial^j U(\mathbf{x}_\perp)$ with $U(\mathbf{x}_\perp) = V_-^\dagger(\mathbf{x}_\perp)V_+(\mathbf{x}_\perp)$ we thus conclude that $\boldsymbol{\alpha}_{\perp a}(\mathbf{x}_\perp)$ must be (in this gauge) of the form

$$\alpha^i(\mathbf{x}_\perp) = -\frac{i}{g} U^\dagger(\mathbf{x}_\perp)\partial^i U(\mathbf{x}_\perp). \tag{10}$$

Equation (10) together with Eq. (8) thus determine $\alpha^i(\mathbf{x}_\perp)$ uniquely (up to some trivial constants).

The above results have a number of applications as we will discuss below. First of all, Eq. (8) alerts us again that in the light-cone gauge one must not assume a vanishing of the gauge fields at $x^- = \pm\infty$. However, the most important application of Eq. (8) lies in the fact that it allows us to reexpress $\boldsymbol{\alpha}_{\perp a}(\mathbf{x}_\perp)$ in terms of other degrees of freedom. The interesting aspect about this observation is the fact that $\boldsymbol{\alpha}_{\perp a}(\mathbf{x}_\perp)$ also appears in the correlation function (6) for the average transverse momentum. In the rest of this paper we will discuss the implication of this fundamental result.

IV. AVERAGE ⊥ MOMENTUM IN QCD

In QCD, the condition that the gauge field at $\pm\infty$ is pure gauge is nonlinear, which prevents us from writing down closed form solutions to the finiteness conditions (8), (10). If one makes the ansatz $U(\mathbf{x}_\perp) = e^{-ig\phi_a(\mathbf{x}_\perp)\lambda_a/2}$ then, to lowest order in ϕ_a, one finds the QED-like condition $\Delta\phi_a(\mathbf{x}_\perp) = -\rho_a(\mathbf{x}_\perp)$, yielding

$$\alpha_a^i(\mathbf{x}_\perp) = -\partial^i\phi_a = -\int \frac{d^2\mathbf{y}_\perp}{2\pi} \frac{x^i - y^i}{|\mathbf{x}_\perp - \mathbf{y}_\perp|^2} \rho_a(\mathbf{y}_\perp). \tag{11}$$

Of course, there are non-Abelian corrections to this result and therefore Eq. (11) is not an exact solution to the finiteness conditions in QCD. However, since we were unable to find an exact operator solution, we will proceed using Eq. (11).

Upon inserting Eq. (11) into Eq. (6) one obtains

$$\langle k_q^i \rangle = -\frac{g}{4p^+} \int \frac{d^2\mathbf{y}_\perp}{2\pi} \frac{y^i}{|\mathbf{y}_\perp|^2} \langle p|\overline{q}(0)\gamma^+ \frac{\lambda_a}{2} q(0)\rho_a(\mathbf{y}_\perp)|p\rangle. \tag{12}$$

The physical interpretation of this result is that the average transverse momentum of quarks of flavor q can be related to correlations on the transverse plane. The specific correlations that appear in Eq. (12) reflect a Coulomb interaction between the active quark and the spectators. Here a Coulomb interaction appears because we have solved the finiteness constraints in QCD only to first order. Equation (12) is thus equivalent to treating the FSI in lowest order in perturbation theory [5,12,14,15]. Note also that the resulting correlation functions are very similar to the correlation functions that have been used to describe the small-x gluon distributions in nuclei [17].

To first order, what we also find is that the average transverse momentum due to the FSI of all constituents (quarks +gluons) added together vanishes for symmetry reasons. It is not clear if this happens beyond lowest order.

Equation (12) may be useful for several reasons. While the original expression for the Sivers asymmetry involved a gauge link, which made a parton model interpretation difficult, Eq. (12) does have an immediate parton model interpretation in terms of color-flavor correlations in the transverse plane. This may be useful in correlating experimental data with our understanding of the nucleon structure. Another use of Eq. (12) is that it can be directly calculated from the light-cone wave functions of the nucleon. Of course, we need to keep in mind that (unlike the QED case) Eq. (12) is only an approximation, but we still believe that this result provides a step toward linking the Sivers asymmetry with other features of hadron structure. Finally, we would like to emphasize that Eq. (12) suggests interesting connections between the distribution of partons in impact parameter $(\mathbf{x}_\perp)$ and the sign of the transverse SSA [16]. For example, in a simple quark model, such as the bag model [12], the color part of the matrix element in Eq. (12) would be negative (attraction). If the transverse distribution of j_q^+ is transversely shifted relative to the spectators [18] then the resulting average transverse momentum has the opposite sign to the sign of the transverse distortion in impact parameter space.

V. SUMMARY

We have studied the average transverse momentum of gauge invariant quark distributions for a transversely polarized target in the light-cone gauge. The Wilson line is along the light cone to infinity to incorporate the final state interactions in semi-inclusive DIS. In the light-cone gauge, the Wilson-line phase factor receives its only nonzero contribution from the gauge field at $x^- = \pm\infty$. In a naive Fock space expansion the gauge field at $x^- = \pm\infty$ is usually implicitly set to zero, thus making a correct treatment of single-spin asymmetries rather difficult (except in perturbation theory, where one can carefully regularize the fields at $x^- = \pm\infty$ "by hand").

We have also studied conditions for the infrared ($x^- = \pm\infty$) convergence of the light-cone Hamiltonian for gauge theories and derived operator conditions that need to be satisfied in order for the light cone to be free of infrared divergences arising from otherwise ill-defined operators $1/i\partial_-$. This operator condition relates the transverse component of the gauge field $A_a^i(\pm\infty^-,\mathbf{x}_\perp)$ to the color density $\rho_a(\mathbf{x}_\perp)$ integrated over all x^-.

Fortuitously, the same kind of operator that governs the average transverse momentum in gauge invariant quark distributions also appears in the finiteness conditions for the light-cone Hamiltonian. We are thus able to eliminate $A_a^i(\pm\infty^-,\mathbf{x}_\perp)$ in the average transverse momentum in favor of other, less infrared singular, degrees of freedom. In QED we can solve the operator condition arising from finiteness conditions exactly and we are able to express the average transverse momentum in terms of charge density correlations in the transverse plane. In QCD we were only able to solve the operator condition to first order in the color charge density and there we find a similar result as in QED, namely that the average transverse momentum can be related to transverse correlations between the active quark and the spectators.

Single spin asymmetries do not have a simple parton model (or light-cone Fock space) interpretation. The main significance of our results is that we have found relations that allow us to relate the average transverse momentum to operators that do have a parton interpretation (in QED exactly, in QCD approximately). One immediate application of these results is that it allows us to evaluate the average transverse momentum of the quarks directly from the nucleon wave function in light-cone quark models.

Several extensions of this work are conceivable. First it would be desirable to derive an exact solution (at least in terms of an expansion) for the finiteness conditions in QCD, so that one can study the effects of higher order terms that we have omitted. Second, it would be interesting to see if one can translate the results from the work into lattice language (Euclidean as well as transverse lattice) with the goal of being able to compute the average transverse momentum nonperturbatively within these frameworks.

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